

Article ID: 1000 8055(2007) 04 0645-08

Applications of numerical optimization techniques to design of axial compressor blades

Choor Man Jang¹, Kwang Yong Kim^{2*}

(1. Fire and Engineering Services Research Department, KICT, Rep. of Korea;

2 Department of Mechanical Engineering, Inha University, Rep of Korea)

Abstract: This paper describes the shape optimization of NASA rotor 37 and rotor and stator blades in a single stage transonic axial compressor. Shape optimization of the blades operating at the design flow condition has been performed using the response surface method and three dimensional Navier Stokes analysis. Third layer approximation is introduced to the Navier Stokes equations, and an explicit Runge Kutta scheme is used to solve the governing equations. The three design variables, blade sweep, lean and skew, are introduced to optimize the three dimensional stacking line of the blades. The objective function of the shape optimization is an adiabatic efficiency. Throughout the optimization of rotor and stator blades, optimal blade shape can be obtained. It is noted the increase of adiabatic efficiency by optimization of the blade shape with the stacking line in the single stage transonic axial compressor is more effective in a rotor blade rather than a stator blade because of the large deformation of blade shape in the stator blade.

Key words: aerospace propulsion system; axial compress; numerical optimization; sweep; lean; skew

CLC number: V231.3

Document code: A

Introduction

This paper describes the shape optimization of a transonic axial compressor rotor (NASA rotor 37) and a rotor blade in a single stage transonic axial compressor. The design method of axial flow compressors based on the three dimensional flow analysis has been developed remarkably in recent years with the rapid enhancement of computational power. The accurate flow prediction inside a transonic axial compressor rotor using computational fluid dynamics (CFD) is difficult due to its extremely complex features: three dimensional, unsteady and vortical nature of flow in the blade passage^[1-3]. However, CFD has obvious advantages compared to the traditional experimental analysis. CFD helps us to analyze the effects of individual feature more easily as compared to the experimental method.

Recently, the use of sweep, lean (dihedral),

and skew (stacking line in rotational direction) in an axial flow compressor rotor has become a matter of interest in the design of turbomachinery blades^[4-10]. These blade shape parameters, which form a three dimensional stacking line, are generally introduced to reduce shock losses, corner separation in the blade hub, and tip clearance losses in transonic compressor rotors. For example, Gallimore et al.^[4] introduced three dimensional blade designs using a sweep and a lean in an axial flow compressor rotor for engine. They showed that the positive lean reduces the hub corner and tip clearance losses except near the mid-span region. They also showed that the improvement in the compressor performance as well as the large reduction in the cost and time-scales associated with a rig test was obtained together with CFD calculation.

One of the significant design trends is the use of aerodynamic sweep to improve the per-

formance and stability of transonic compressor blades. The pioneer study on blade sweep in compressors has been done by Bliss^[5]. The main objective of this study was to reduce the noise level induced by shock waves. Hah, et al.^[6] studied both forward and backward swept compressor blades and showed that a backward swept blade suppressed the intensity of the shock loss and a forward swept blade suppressed secondary flow and tip entropy generation. Watanabe and Zangeneh^[7] reported that the blade sweep in the design of a transonic turbomachinery blade was an effective parameter to control the strength and position of the shock wave at the tip of the transonic rotors. Denton and Xu^[8] investigated the effects of sweep and lean on the performance of a transonic fan and showed that the stall margin was significantly improved with the forward swept blade although a very little change in the peak efficiency was produced by the blade sweep or lean.

A number of studies have been done on the advantages of a skewed rotor. Cai, et al.^[9] studied on aerodynamic and aeroacoustic characteristics of an axial flow fan with skewed blade. With the reduction of a secondary flow and the thinness of a rotor wake, they reduced a broadband noise. Fischer, et al.^[10] observed the effect of bowed stators on the performance of a compressor and showed that the separation was reduced in the bowed stator leading to increase in the stagnation pressure ratio and efficiency.

On the other hand, response surface method (RSM)^[11,12], which is a global optimization method, is recently introduced as a tool of design optimization in turbomachinery. Ahn and Kim^[13] used response surface method to optimize the stacking line of axial compressor rotor blades. Jang and Kim^[14] performed the shape optimization of stator blade in a single stage transonic axial compressor by using RSM. They showed that the increase in adiabatic efficiency by optimization of the stacking line of a stator blade was caused mainly by suppression of flow separation on the blade suction surface.

In the present study, the response surface method (RSM) combined with three dimensional Navier-Stokes solver is introduced to find optimum blade shape in the transonic axial compressor rotor and stator. Three design variables, blade sweep, lean and skew, are used to optimize three dimensional stacking line of the rotor blade. An adiabatic efficiency is selected as an objective function. The optimal values of each variable are obtained by making a response surface.

1 Test axial compressor

Two kinds of axial compressor are used to optimize the blade shape. NASA rotor 37^[15], an axial flow compressor rotor having a low aspect ratio, is used for blade shape optimization. The detailed specifications of the compressor are summarized in Table 1. The rotor tip clearance is 0.356 mm (0.45 percent span). The measured choking mass flow rate is 20.93 kg/s, which corresponds to 103.67% of the design flow rate. The meridional view of the axial compressor is shown in Fig. 1.

Table 1 Design specifications of NASA rotor 37

Mass flow/(kg/ s)	20.19
Rotational speed/(r/ m in)	17188.7
Pressure ratio	2.106
Inlet hub/ tip ratio	0.7
Inlet tip relative Mach no.	1.4
Inlet hub relative Mach no.	1.13
Tip solidity	1.288
Rotor aspect ratio	1.19
Number of rotor blades	36

On the other hand, a single stage transonic axial compressor, which is the first stage of a two stage axial transonic compressor designed for a helicopter gas turbine engine^[16], is considered to optimize the blade shape. The detailed specifications of the compressor are summarized in Table 2. The relative Mach number at the rotor tip is 1.177, and the stage pressure ratio is 1.506. The rotor tip clearance is 0.4 mm, and the maximum diameter of the rotor tip is 176 mm.

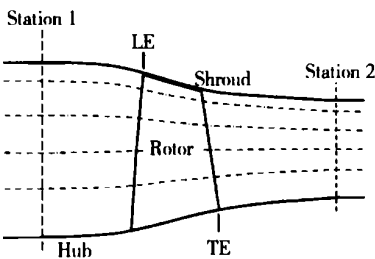


Fig. 1 Meridional view of Rotor 37

The measured choking mass flow rate is 3. 235 kg/s, which corresponds to 106 4% of the design flow rate. The meridional view of the single stage axial compressor is shown in Fig. 2. The compressor has a total axial length of 137. 5 mm.

Table 2 Design specifications of a single stage axial compressor

Flow rate/(kg/s)	3. 04
Rotational speed/(r/min)	41500
Pressure ratio	1. 506
Rotor tip speed/(m/s)	382
Adiabatic efficiency/ %	86. 3
Number of rotor blades	17
Number of stator blades	26

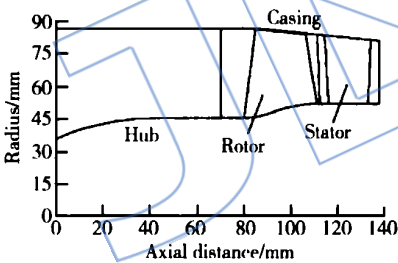


Fig. 2 Meridional view of a single stage axial compressor

2 Method of blade optimization and numerical analysis

2.1 Method of blade optimization

Optimization using RSM is a series of statistical and mathematical processes i. e. , generation of data by numerical computations or experiments, construction of response surface by interpolating the data, and optimization of the objective function on the surface. The overall procedures of the RSM-based optimization are shown in Fig. 3. At first, an objective function is defined

as a primary parameter for the shape optimization. Design variables are selected among the variables for which objective function is sensitive, and then experimental points are determined with the help of design of experiments (DOE).

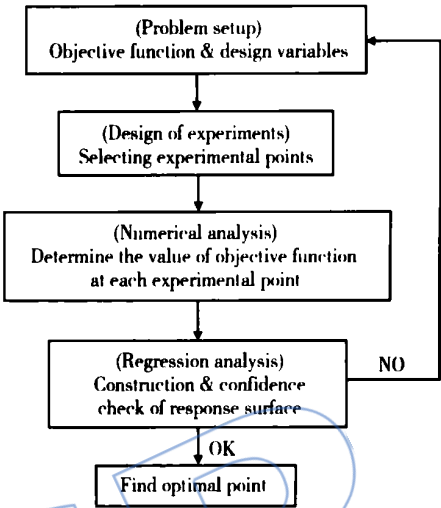


Fig. 3 Optimization procedure using RSM

In the next step, the value of objective function at each experimental point is obtained by numerical analysis or experiment. Finally, a response surface is determined to obtain an optimal point. Although RSM was devised to obtain empirical correlation from the experimental data but the ability to reduce the number of experiments let this method be applied widely to the optimization problems^[11].

The polynomial-based response surfaces are commonly employed in RSM. In the present study, a response model f is assumed as a second-order polynomial, which can be written as follows:

$$f = \beta_0 + \sum_{j=1}^n \beta_j x_j + \sum_{j=1}^n \beta_{jj} x_j^2 + \sum_{i \neq j} \beta_{ij} x_i x_j$$

(1)

where n is the number of design variables, and x and β indicate the design variables and the regression coefficients, respectively. The number of coefficients (β_0, β_1 , etc.) is $(n+1) \cdot (n+2)/2$. Unknown coefficients of polynomial are obtained from a standard least-squares regression. In this case, the number of data must be larger than the number of coefficients.

2.2 Objective function and design variables

In the present study, adiabatic efficiency η_{ad} is selected as an objective function for the shape optimization of the rotor blades, and is defined as follows:

$$\eta_{ad} = \frac{(p_{0exit}/p_{0inlet})^{(\gamma-1)/\gamma} - 1}{T_{0exit}/T_{0inlet} - 1} \tag{2}$$

where p_0 and T_0 are total pressure and total temperature, respectively.

To enhance the adiabatic efficiency of the compressor rotor (NASA rotor 37), three dimensional blade stacking line is optimized by introducing three shape variables; namely sweep, lean, and skew as shown in Figs. 4-6.

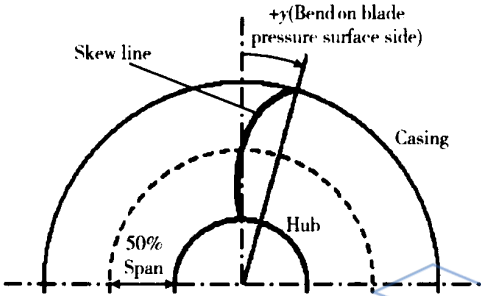


Fig. 4 Definition of blade skew (front view)

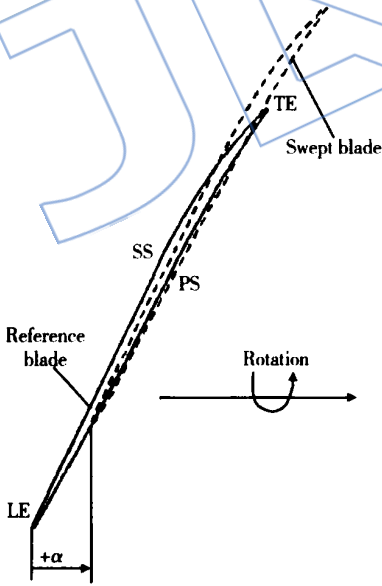


Fig. 5 Definition of blade sweep

A variety of terms have been used to define the stacking line (e. g. sweep, lean, skew etc.) and no consistent nomenclature has emerged. The skew angle defined at rotor tip, γ in Fig. 4, is taken to be positive if the skew line of the rotor

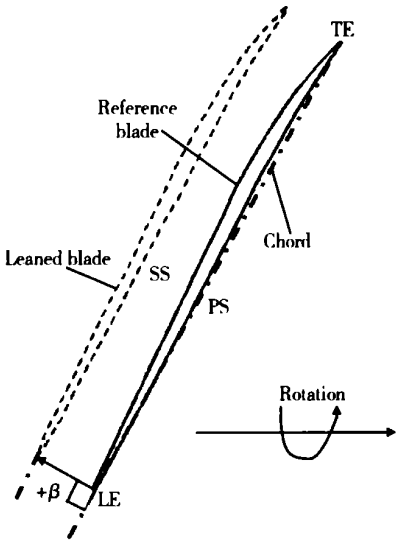


Fig. 6 Definition of blade lean

tip bends to the blade pressure surface side. The skew line is formed by a second order polynomial curve. To complete the skew line, skew angles at hub and at 50 percent span are set to zeros.

The term of sweep is used to describe movement of airfoil section in the manner shown in Fig. 5. That is, movement parallel to the airfoil chord line is termed sweep. A sweep value, α in Fig. 5, is defined at the rotor tip and normalized by the axial tip chord ($= 27.77 \text{ mm}$). The blade sweep is taken to be positive if the airfoil sections are moved in the downstream direction. The line of the swept blade between the rotor tip (α in Fig. 3) and hub ($= \text{zero}$) is linearly connected while the gap of the tip clearance is kept constant.

Figure 6, which is top view of the rotor, shows the definition of blade lean (or dihedral). Lean is defined when the blade moves normal to the airfoil chord line. The lean value, β in Fig. 6, is defined at the rotor tip and normalized by the axial tip chord. This definition is similar to that of the blade sweep. The blade lean is taken to be positive if the airfoil sections are moved to the blade suction side. The value of blade lean is zero at hub, and is linearly connected to the rotor tip keeping constant tip clearance. The range of each variable for selection of the points for response evaluation is determined by preliminary calcula-

tions, and is summarized in Table 3.

Table 3 Design space of blade sweep, lean, and skew

Variables	Lower Bound	Middle	Upper Bound
Sweep(α) / %	0.0	12.6	25.2
Lean(β) / %	- 3.6	- 1.8	0.0
Skew (γ) / rad.	0.0	0.05	0.1

On the other hand, a stacking line of the rotor and stator blades is introduced to enhance the adiabatic efficiency of the rotor blade (single stage axial compressor). In the present study, the stacking line is defined by two shape variables, a skew angle at the mid-span section, θ and a skew angle at the tip, δ of the rotor and stator blades as shown in Fig. 7. The angle has a plus value when the stacking line bends on the blade pressure surface side. Throughout preliminary study, skew angles at the mid-span (θ) are determined by -0.37 and -0.06 for the rotor and stator blades, respectively. The ranges of skew angles for selection of the points for response evaluation are summarized in Table 4.

Table 4 Design space of stacking line

Blades	Angle	Lower	Middle	Upper
Rotor	θ		-0.37	
	δ	0.08	0.19	0.3
Stator	θ		-0.06	
	δ	0.35	0.5	0.65

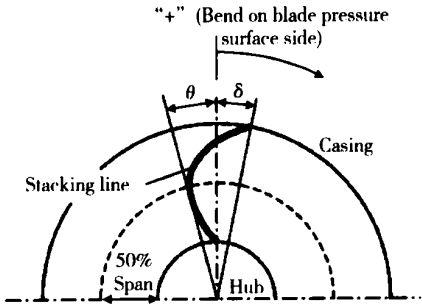


Fig. 7 Stacking line for optimization of rotor blade

2.3 Numerical analysis method

The three-dimensional thin-layer Navier-Stokes and energy equations are solved on body-fitted grids using an explicit finite-difference scheme. An explicit Runge-Kutta scheme pro-

posed by Jameson, *et al.*^[17] is used to solve flow from initial to steady state with a spatially varying time step to accelerate convergence. Artificial dissipation terms have been added to resolve shocks. The algebraic turbulence model of Baldwin and Lomax^[18] has been employed to estimate the eddy viscosity.

Figure 8 shows the perspective view of computational grid for the NASA rotor 37 and the single stage transonic axial compressor.

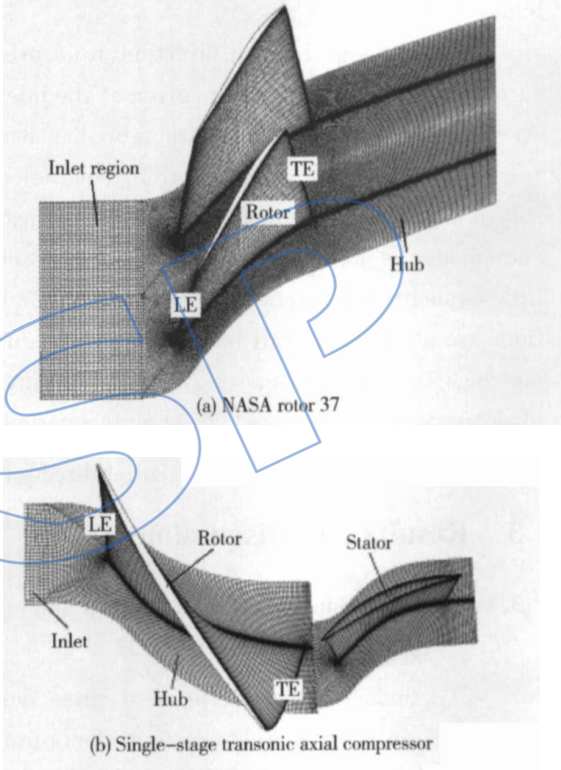


Fig. 8 Perspective view of computational grid

For the NASA rotor 37, a composite grid system with structured H-, G-, and O-type grids is adopted to represent the complicated configuration of the axial compressor. H-type grid consists of $60 \times 36 \times 63$ grids (in the streamwise, pitchwise and spanwise directions, respectively), and is introduced for the inlet flow region. G-type grid consists of $350 \times 46 \times 63$ grids, and is used for the blade passage. The O-type grid embedded in the tip clearance consists of $182 \times 13 \times 13$ grids. Grid spacing for the first grid point from the blade surface gives $y^+ < 5.0$. The whole grid system has about 1 181 000 grid points. The average number of iterations and CPU time for the converged solution are approximately 3 000

and 3.5 hours with supercomputer of NEC SX-6 (144 GFLOPS), respectively.

For the single-stage transonic axial compressor, H-type grids for the inlet flow region and O-type grids for the tip clearance region consist of $50 \times 63 \times 32$ (in streamwise, spanwise and pitchwise directions) and $162 \times 12 \times 12$ grids, respectively. G-type grids for the rotor and the stator consist of $178 \times 63 \times 38$ and $120 \times 63 \times 41$ grids, respectively. The whole grid system has about 860 000 grid points.

Mach numbers in each direction, total pressure and total temperature are given at the inlet. At exit, the hub static pressure ratio has been specified and the radial equilibrium equation is solved along the blade span. A periodic tip clearance model is used to resolve the tip clearance flow explicitly. No-slip and adiabatic wall conditions are used at the wall boundaries. For reducing the computational load, flow field in a single blade passage is simulated by applying periodic boundary condition in the tangential direction.

3 Results and discussion

3.1 Optimization of blade lean, sweep and skew of NASA rotor 37

To understand the effects of three design variables on the objective function, the optimization has been done by coupling the variables like lean sweep, skew-lean, and sweep-skew. As described for the design space in Table 3, nine training points are necessary for each coupled case having three boundary levels. Seven training points, however, are selected in the present calculation by D-optimal design. After determining the values of objective function (adiabatic efficiency) for each training point, a response surface is constructed. The polynomial-based response surface having a second-order is used.

Results of the blade shape optimization for the three-coupled variables at the design flow rate are shown in Table 5. The model of lean sweep-skew is determined by using the mean value of optimized three design variables as

shown in Table 5. In the present calculation, the mean values of optimal sweep, lean and skew is 10.26, -1.48 and 0.065 , respectively.

Table 5 Optimum value of design variables

Design variables	Lean sweep	Skew-lean	Sweep-skew	Mean value
Sweep(α) / %	10.08	-	10.44	10.26
Lean(β) / %	-1.60	-1.35	-	-1.48
Skew(γ) / rad	-	0.067	0.064	0.065

Results of the blade shape optimization for the three-coupled variables at the design flow rate are shown in Table 6. The model of lean sweep-skew is determined by using the mean value of optimized three design variables as shown in Table 5. Adiabatic efficiency, which is the objective function of the present optimization, is successfully increased for three coupled cases. Figure 9 shows the meridional blade shape of the reference and the optimum blade shape. As shown in the figure, the optimal shape of the rotor blade has a backward sweep.

Table 6 Results of optimization for the adiabatic efficiency(%)

Model	Reference/ %	Optimized shape/%	Increment/ %
Lean sweep	88.65	89.5	0.95
Skew-lean		89.9	1.41
Sweep-skew		89.7	1.18
Lean sweep skew		89.9	1.41

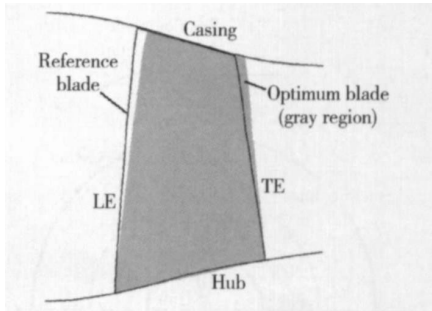


Fig. 9 Comparisons of reference and optimum blades (meridional view)

3.2 Optimization of the rotor and stator blades in the axial compressor

The blade optimization in a single-stage transonic axial compressor is performed using RSM and numerical analysis.

The resulting response surface is shown in Fig. 10. In this figure, horizontal and vertical axes represent the design variables of δ_{Rotor} and δ_{Stator} , respectively. The optimum design variables, δ_{Rotor} and δ_{Stator} are 0.13 and 0.56, respectively. The optimum position having maximum adiabatic efficiency is located inside the boundary of design space as shown in Table 4. It is noted that the angle of the stacking line at stator δ_{Stator} is so large, thus resulting in large deformation of a stator blade.

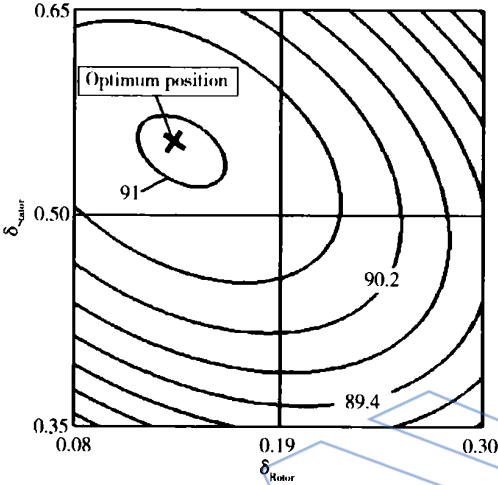


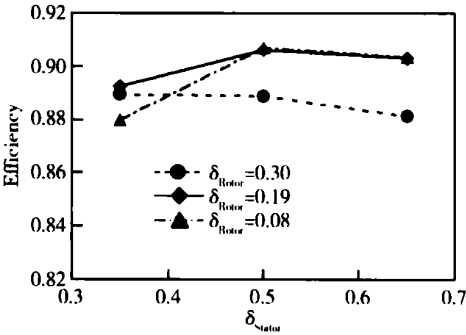
Fig. 10 Response surface for adiabatic efficiency % (contour intervals= 0.4)

Table 7 shows the result of optimization at the design flow rate. Adiabatic efficiency of the compressor, which is the objective of this optimization, is successfully increased by 3.75 percent.

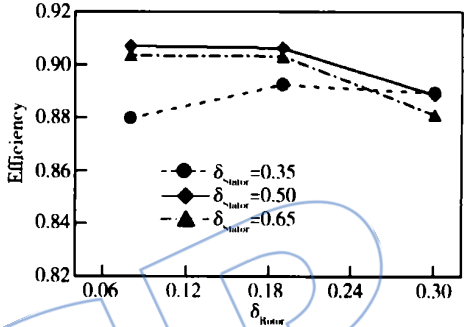
Table 7 Results of optimization

	Initial shape	Optimized shape	Increment/%
A diabat ic efficiency / %	88.0	91.3	3.75

Fig. 11 shows the distributions of efficiency for design variables. To understand the effects of each design variable on the efficiency, the efficiency is obtained at the nine training points. The optimum design variables, δ_{Rotor} and δ_{Stator} are observed at 0.19 and 0.5, respectively, which corresponds well to the value obtained by response surface as shown in Fig. 10.



(a) Effect of stator angle on the rotor angle



(b) Effect of rotor angle on the stator angle

Fig. 11 Distributions of efficiency for design variables

4 Conclusion

The shape optimization of rotor blades for transonic axial compressors has been performed by the response surface method and the three-dimensional Navier-Stokes analysis. The adiabatic efficiency is successfully increased by reducing the losses by optimizing the three-dimensional stacking line with lean, sweep and skew for the NASA rotor 37. In the shape optimization of rotor and stator blades of a single-stage transonic axial compressor, the adiabatic efficiency is increased by 3.75 percent as compared to that of the reference shape. It is found that the increase of adiabatic efficiency by optimization of the blade shape with the stacking line is more effective in a rotor blade rather than a stator blade because of the large deformation of blade shape in the stator blade.

References

[1] Suder K L, Celestina K L. Experimental and computational investigation of the tip clearance flow in a transonic axial compressor rotor[R]. Proceedings of ASME Paper 94 GT-

- 365.
- [2] Hah C, Rabe D C, Wadia A R. Role of tip leakage vortices and passage shock in stall inception in a sweep transonic compressor rotor [R]. Proceedings of ASME GT-2004 53867.
- [3] Chima R V. Calculation of tip clearance effects in a transonic compressor rotor [J]. ASME Journal of Turbomachinery, 1998, 120(1): 131-140.
- [4] Gallimore S J, Bolger J J, Cumpsty N A. The use of sweep and dihedral in multistage axial flow compressor blading, part 1: university research and methods development [R]. Proceedings of ASME GT-2002 30328.
- [5] Bliss D B. Method of and Apparatus for preventing leading edge shock related noise in Transonic and supersonic blades and like [P]. US Patent 3989406, 1976.
- [6] Hah C, Puterbaugh S L, Wadia A R. Control of shock structure and secondary flow field inside transonic compressor rotor through aerodynamic sweep [R]. Proceedings of ASME 99-GT-561.
- [7] Watanabe H, Zangeneh M. Design of the blade geometry of swept transonic fans by 3D Inverse design [R]. Proceedings of ASME Turbo Expo, GT-2003 38770.
- [8] Denton J D, Xu L. The effects of lean and sweep on transonic fan performance [R]. Proceedings of ASME Turbo Expo, GT-2002 30327.
- [9] Cai N, Xu J, Benaissa A. Aerodynamic and aeroacoustic performance of a skewed rotor [R]. Proceedings of ASME Turbo Expo, GT-2003 38592.
- [10] Fischer A, Riess W, Seume J. Performance of strongly bowed stators in a 4 stage high speed compressor [R]. Proceedings of ASME Turbo Expo, GT-2003 38392.
- [11] Myers R H, Montgomery D C. Response surface methodology: process and product optimization using designed experiments [M]. John Wiley & Sons, New York, 1995.
- [12] Sevant N E, Bloor M I G, Wilson M J. Aerodynamic design of a flying wing using response surface methodology [J]. J. Aircraft, 2000, 37: 562-569.
- [13] Ahn C S, Kim K Y. Aerodynamic design optimization of a compressor rotor with Navier-Stokes analysis, proceedings of the institution of mechanical engineers [J]. Part A: Journal of Power and Energy, 2003, 217(2): 179-184.
- [14] Jang C M, Kim K Y. Optimization of a stator blade using response surface method in a single stage transonic axial compressor, proceedings of the institution of mechanical engineers, part A [J]. Journal of Power and Energy, 2005, 219(8): 595-603.
- [15] Reid L, Moore R D. Design and overall performance of four highly-loaded, high speed inlet stages for an advanced [R]. High Pressure Ratio Core Compressor, 1978, NASA TP-1337.
- [16] Kim C T, Kim J H, Lee D S. Performance test of a two stage axial compressor of a turboshaft engine for helicopter [R]. Proceedings of the KSAS Spring Conference, 1999, 327-331.
- [17] Jameson A, Schmidt W, Turkel E. Numerical solutions of the euler equation by finite volume methods using runge-kutta time stepping schemes [R]. AIAA Paper No. 81-1259, 1981.
- [18] Baldwin B S, Lomax H. Thin layer approximation and algebraic model for separated turbulent flow [R]. AIAA Paper No. 78-257, 1978.